PERGAMON

International Journal of Heat and Mass Transfer 45 (2002) 2629–2632

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Technical Note

Parametric study of transient free convection heat transfer

Thierry de Lorenzo, Jacques Padet *

U.T.A.P., Thermomécanique, E.A. 2061, Faculté des Sciences, B.P. 1039, 51687 Reims, France

Received 22 December 2000; received in revised form 25 July 2001

1. Introduction

In several situationss the control of a surface temperature is ensured by convective exchange; for examples for steel metallurgy or electronic components. And among the three types of convective exchanges, forced convection is often used because of its efficiency. *A contrario* natural convection has the advantage to be free in terms of energy expense but generates low heat transfer coefficient. Thus it will be interesting to improve free convection heat transfer in order to substitute for the expensive forced convection.

Laminar free convection problem on a vertical wall has been plentifully investigated considering a wall heat flux density constant [1–3] or varying sinusoidally [4]. But only two studies deal with the influence of the different parameters of the problem. Thereby Yang et al. [4] evaluated the influence of the amplitude and of the frequencies of the sinusoid whereas the Vargas and Bejan [5] study is based on the optimisation of heat transfer.

This last investigation considers a fluid (water) initially at rest before being suddenly put in motion thanks to a periodical heat flux density applied to the wall. Scale analysis, numerical and experimental studies have been carried out and show that the heat transfer is optimised when the heating and cooling (wall adiabatic) periods are equal.

In this paper the wall is subjected to a uniform heat flux density until the steady states (thermal and dynamical) are reached. Then a periodical heat flux density which varies from q_w to 0 (adiabatic wall) is applied to the surface in order to improve the free convection heat transfer. A parametric study is then carried out in order to optimise the heat transfer.

* Corresponding author. Tel.: +33-3-2691-8310; fax: +33-3-2691-8310.

E-mail address: jacques.padet@univ-reims.fr (J. Padet).

2. Modelling

Consider that the fluid in contact with the vertical surface has initially reached its steady states (thermal and dynamical). At time $t = 0$ the wall is subjected to a uniform periodical heat flux density $q_w(t)$, which can be divided into several identical cycles. During one cycle, e.g. from $t = 0$ to t_c (Fig. 1), the wall is supposed to be:

- adiabatic (e.g. $q_w = 0$) during the dimensionless period P_a,
- heated (heat flux density q_w) during P_h.

According to Cebeci [6] in transient regime the velocity profiles are nonsimilar, nevertheless differential method can be used for convenience in numerical work; the dependence on x is not eliminated but reduced.

The use of the differential method implies the existence of a stream function ψ, defined such as $U = \partial\psi/\partial y$ and $V = -\partial\psi/\partial x$. The dimensionless temperature and velocity expressions must then be considered varying with two variables in space (X, η) and one in time (t^+), as follows:

$$F'(X, \eta, t^+) = \frac{U}{U^0(x)} = \frac{x}{\nu Gr_x^{*2/5}} U$$

and

$$\theta(X, \eta, t^+) = \frac{\lambda Gr_x^{*1/5}}{q_w x}(T - T_\infty), \tag{1}$$

where $Gr_x^* = g\beta q_w x^4/\lambda\nu^2$ is the modified Grashof number, and X, η and t^+ are obtained using the classical dimensionless transformed variables of the differential method (see [3]):

$$X = \frac{x}{L}, \quad \eta = \frac{y}{x} Gr_x^{*1/5}, \quad t^+ = \frac{U^0(x)}{x}, \quad t = \frac{\nu t}{x^2} Gr_x^{*2/5}. \tag{2}$$

Laminar boundary layer system of equations [7] becomes using the dimensionless variables (1) and (2):

0017-9310/02/$ - see front matter © 2002 Published by Elsevier Science Ltd.
PII: S0017-9310(01)00351-9

Nomenclature

c	cycle number
h	heat transfer coefficient, W m^{-2} K^{-1}
h^+	dimensionless heat transfer coefficient
n	period rate
q	heat flux density, W m^{-2}
t^+	dimensionless time
D_T	transient duration, S
Gr^*	modified Grashof number
L	plate length, M
P	period
Pr	Prandtl number
T	temperature, K
U, V	longitudinal and orthogonal velocities, M
U^0	velocity reference, m s^{-1}
X	dimensionless longitudinal coordinate

Greek symbols

β	fluid dilatability, K^{-1}
η	dimensionless orthogonal coordinate
λ	thermal conductivity, W m^{-1} K^{-1}
ν	kinematic viscosity, m ^{2} s^{-1}
θ	dimensionless temperature
ψ	stream function

Subscripts

a	adiabatic
h	heat
w	wall
∞	ambient fluid

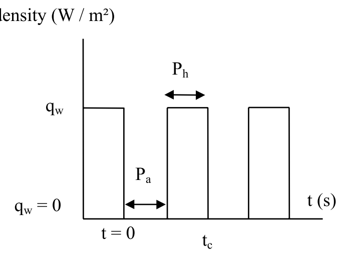

Fig. 1. Periodical heat flux density.

$$F''' + \frac{4}{5}FF' - \frac{3}{5}\left(F'\right)^2 + \theta = \frac{\partial F'}{\partial t^+} + X\left[F'\frac{\partial F'}{\partial X} - F''\frac{\partial F}{\partial X}\right],$$
$$\frac{\theta''}{\mathrm{Pr}} + \frac{4}{5}F\theta' - \frac{1}{5}F'\theta = \frac{\partial \theta}{\partial t^+} + X\left[F'\frac{\partial \theta}{\partial X} - \theta'\frac{\partial F}{\partial X}\right]. \qquad (3)$$

In equations above the symbol "′" denotes differentiation with respect to η.

At $t^+ < 0$, the flow has reached its steady states (thermal and dynamical) while the dimensionless boundary conditions are when $t^+ \geqslant 0$:

$$X \geqslant 0: \qquad F = F' = 0, \quad \theta' = \text{constant} \quad \text{on } \eta = 0,$$
$$F' = \theta = 0 \quad \text{when } \eta \to \infty. \qquad (4)$$

System of Eq. (3) subjected to the boundary conditions (4) is resolved using the implicit, iterative, tri-diagonal finite-difference method known as the Keller-box method. This method has proven to be successful to get accurate results to resolve two-dimension, three-dimension, steady and transient systems [6,8,9]. Moreover, as Cebeci advocates the box method removes the singularity that the physical coordinates have at $x = 0$ [9].

This method has been validated for this problem by comparing transient values with results known as the reference ones [10], in a step heat flux density problem (see [3]).

A nonuniform grid distribution (Tchebyshev) in the η direction with a small initial step size is used to accommodate steep changes in the velocity and temperature gradients in the immediate vicinity of the wall. In the X and t^+ directions, uniform distribution grid has been used.

A convergence criterion based on the relative difference between the current and the previous iterations is utilised. When this difference reaches 10^{-5}, the solution is assumed converged and the iteration procedure is terminated.

3. Parametric study

3.1. Boundary layer conditions verification

To verify if the boundary layer conditions, used to simplify the free convection governing equations, are still correct when periodical heat flux density is applied to the wall, comparison between the different terms of system (3) has been done.

Thereby by comparing the velocities U, and V, their derivates and the different temperature gradients, we note that the boundary layer conditions are valid if the fluid considered has $Pr = O(1)$ or $Pr \ll 1$, e.g. when the thermal boundary layer thickness is inferior or equal to the dynamical one.

On the other hand for more viscous fluids, e.g. $Pr \gg 1$, the longitudinal velocity U is still higher than the orthogonal one V but not enough to consider $U \gg V$.

Therefore only the case where the fluid in contact with the surface is air ($Pr = 0.7$) is considered in this paper.

3.2. Effect of the period

The computations have been carried out for a modified Grashof number equal to 1.01×10^7 in order to let

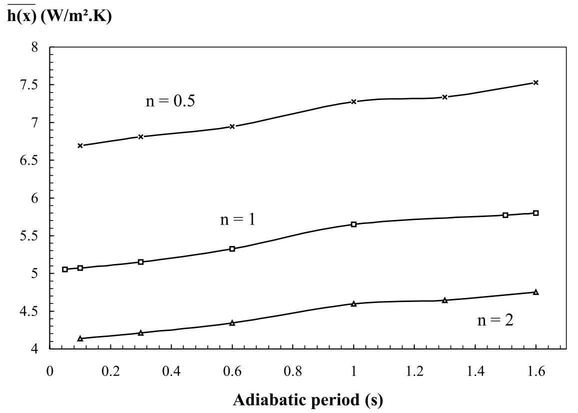

Fig. 2. Average heat transfer coefficient $\overline{h}$ vs adiabatic period for different period rates n, $Pr = 0.7$, $Gr_x^* = 1.01 \times 10^7$.

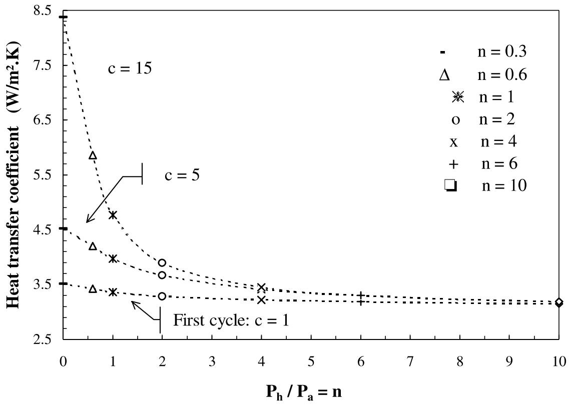

Fig. 3. Heat transfer coefficient vs period rates n after different cycles heat flux c, $Pr = 0.7$, $Gr_x^* = 1.01 \times 10^7$.

the regime flow laminar. Moreover adiabatic and heated periods are linked such as:

$$P_{\mathrm{h}} = nP_{\mathrm{a}}. \tag{5}$$

At last to distinguish easily any heat transfer improvement due to periodical heat flux density, the heat transfer coefficient has been averaged on time from $t^+ = 0$ to D_T, as follows:

$$\overline{h(x)} = \frac{1}{D_T}\int_0^{D_T} h(x,t)\,\mathrm{d}t, \tag{6}$$

where D_T is chosen equal to 10.25 s to let the flow reach its steady state whatever the period applied to the wall.

As can be seen in Fig. 2, for the same adiabatic period P_{a}, the heat transfer improves when the period rate n gets lower. When the heating period is half of the adiabatic one ($n = 0.5$) the average heat transfer increases by 35%, whereas for $n = 2$ heat transfer decreases from about 20%.

Not shown in Fig. 2 is that for $n = 1$ and $P_{\mathrm{a}} \to 0$ the average heat transfer $\overline{h}$ tends to the steady value $\overline{h} = 4.25$ W m^{-2} K^{-1}.

Thereby to improve heat transfer you must let time to the surrounding fluid to refresh itself during the adiabatic period before applying a new heating period P_{h} at the wall ($n \leqslant 1$).

Thus the heat transfer is optimum if the adiabatic period is inferior or equal to the heated one and if the period rate is small.

4. Effect of cycle numbers

Heat transfer coefficient after different cycle numbers and for several period rates n is presented in Fig. 3. Note that Fig. 3 is not a classical $y = f(x)$ representation.

The heat transfer coefficient is optimised when the period rate n is low, and after several heat flux cycles e.g. when the steady state is reached. When period rate n tends to infinity, e.g. $P_{\mathrm{h}} \to \infty$, the cycle numbers have no more influence and the heat transfer coefficient is equal to the steady value $h = 3.21$ W m^{-2} K^{-1} (for uniform heat flux step problem [10]). On the other hand when $n \to 0$, or $P_{\mathrm{h}} \to 0$, and steady state is reached (e.g. $c \to \infty$) h tends to infinity.

Thus the more flow is disrupted by the periodical heat flux density, the more heat transfer coefficient is improved.

5. Conclusion

We have investigated in this paper how free convective heat transfer can be optimised when a periodical heat flux density is applied to a vertical surface. The boundary layer equations, which are valid only for low viscous fluids, are resolved using differential and Keller-box methods. Investigation on the influence of the period rate n, of the adiabatic period P_{a}, and of the cycle numbers c was then carried out.

It was found that for low period rate ($n \leqslant 1$) and large cycle numbers (e.g. when steady state is reached) heat transfer coefficient gets its optimal value. It was also shown that for a same period rate n the average heat transfer coefficient is maximum if the adiabatic period is large.

Thus the heat transfer is optimum when the vertical plate is more refreshed than heated and when the flow is disrupted by the periodical heat flux density.

Acknowledgements

Authors wish to thank the French Network AMETH (Amélioration des Echanges Thermiques) for its financial support.

References

[1] R. Siegel, Transient free convection from a vertical plate, Trans. ASME 80 (1958) 347–359.

[2] B. Gebhart, Natural convection cooling transients, Int. J. Heat Mass Transfer 7 (1964) 479–483.

[3] Th. de Lorenzo, Transfert de chaleur par convection naturelle en régime variable, Ph.D. Thesis, University of Reims-Champagne Ardenne, Reims, France, 2000.

[4] J.W. Yang, C. Scaccia, J. Goodman, Laminar natural convection about vertical plates with oscillatory surface temperature, J. Heat Transfer 96 (1974) 9–14.

[5] J.V.C. Vargas, A. Bejan, Optimisation principle for natural convection pulsating heating, J. Heat Transfer 117 (1995) 942–947.

[6] T. Cebeci, P. Bradshaw, Physical and Computational Aspects of Convective Heat Ttransfer, Springer, New York, 1984.

[7] J. Padet, Principes des Transferts Convectifs, Polytecnica, Paris, 1997.

[8] T. Cebeci, Calculation of unsteady two-dimensional laminar and turbulent boundary layers with fluctuations in external velocity, Proc. R. Soc. London, Ser. A 355 (1977) 225–238.

[9] T. Cebeci, Calculation of three-dimensional boundary layers II. Three-dimensional lows in Cartesian coordinates, AIAA J. 13 (1975) 1056–1064.

[10] E.M. Sparrow, J.L. Gregg, Similar solutions for free convection from a nonisothermal vertical plate, Trans. ASME 80 (1958) 379–386.